

## Rubi 4.16.1.4 Integration Test Results

on the problems in the test-suite directory "1 Algebraic functions"

---

Test results for the 1917 problems in "1.1.1.2 (a+bx)^m (c+dx)^n.m"

Problem 369: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left( -\frac{bx^m}{2(a+bx)^{3/2}} + \frac{mx^{-1+m}}{\sqrt{a+bx}} \right) dx$$

Optimal (type 3, 13 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+bx}}$$

Result (type 5, 92 leaves, 5 steps):

$$\frac{x^m \left(-\frac{bx}{a}\right)^{-m} \text{Hypergeometric2F1}\left[-\frac{1}{2}, -m, \frac{1}{2}, 1 + \frac{bx}{a}\right]}{\sqrt{a+bx}} - \frac{2mx^m \left(-\frac{bx}{a}\right)^{-m} \sqrt{a+bx} \text{Hypergeometric2F1}\left[\frac{1}{2}, 1-m, \frac{3}{2}, 1 + \frac{bx}{a}\right]}{a}$$

---

Test results for the 3201 problems in "1.1.1.3 (a+bx)^m (c+dx)^n (e+fx)^p.m"

Problem 957: Result valid but suboptimal antiderivative.

$$\int (ex)^m (a-bx)^{2+n} (a+bx)^n dx$$

Optimal (type 5, 211 leaves, ? steps):

$$\begin{aligned}
& - \frac{(e x)^{1+m} (a-b x)^{1+n} (a+b x)^{1+n}}{e (3+m+2 n)} + \frac{2 a^2 (2+m+n) (e x)^{1+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{1+m}{2}, -n, \frac{3+m}{2}, \frac{b^2 x^2}{a^2}\right]}{e (1+m) (3+m+2 n)} \\
& \frac{2 a b (e x)^{2+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{2+m}{2}, -n, \frac{4+m}{2}, \frac{b^2 x^2}{a^2}\right]}{e^2 (2+m)}
\end{aligned}$$

Result (type 5, 238 leaves, 11 steps):

$$\begin{aligned}
& \frac{a^2 (e x)^{1+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{1+m}{2}, -n, \frac{3+m}{2}, \frac{b^2 x^2}{a^2}\right]}{e (1+m)} \\
& \frac{2 a b (e x)^{2+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{2+m}{2}, -n, \frac{4+m}{2}, \frac{b^2 x^2}{a^2}\right]}{e^2 (2+m)} + \\
& \frac{b^2 (e x)^{3+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{3+m}{2}, -n, \frac{5+m}{2}, \frac{b^2 x^2}{a^2}\right]}{e^3 (3+m)}
\end{aligned}$$

---

Test results for the 159 problems in "1.1.1.4 (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q.m"

Problem 111: Unable to integrate problem.

$$\int \frac{1}{(a+b x)^{3/2} (c+d x)^{3/2} \sqrt{e+f x} \sqrt{g+h x}} dx$$

Optimal (type 4, 786 leaves, ? steps):

$$\begin{aligned}
& - \frac{2d^3 \sqrt{a+bx} \sqrt{e+fx} \sqrt{g+hx}}{(bc-ad)^2 (de-cf) (dg-ch) \sqrt{c+dx}} - \frac{2b^3 \sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx}}{(bc-ad)^2 (be-af) (bg-ah) \sqrt{a+bx}} + \\
& \frac{2b (a^2 d^2 fh - abd^2 (fg+eh) + b^2 (2d^2 eg + c^2 fh - cd (fg+eh))) \sqrt{c+dx} \sqrt{e+fx} \sqrt{g+hx}}{(bc-ad)^2 (be-af) (de-cf) (bg-ah) (dg-ch) \sqrt{a+bx}} - \\
& \left( 2 \sqrt{fg-eh} (a^2 d^2 fh - abd^2 (fg+eh) + b^2 (2d^2 eg + c^2 fh - cd (fg+eh))) \right. \\
& \left. \sqrt{c+dx} \sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{bg-ah} \sqrt{e+fx}}{\sqrt{fg-eh} \sqrt{a+bx}}\right], -\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right] \right) / \\
& \left( (bc-ad)^2 (be-af) (de-cf) \sqrt{bg-ah} (dg-ch) \sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}} \sqrt{g+hx} \right) - \\
& \frac{4bd \sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}} \sqrt{g+hx} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{bg-ah} \sqrt{e+fx}}{\sqrt{fg-eh} \sqrt{a+bx}}\right], -\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right]}{(bc-ad)^2 \sqrt{bg-ah} \sqrt{fg-eh} \sqrt{c+dx} \sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}
\end{aligned}$$

Result (type 8, 39 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a+bx)^{3/2} (c+dx)^{3/2} \sqrt{e+fx} \sqrt{g+hx}}, x\right]$$

---

Test results for the 34 problems in "1.1.1.5 P(x) (a+bx)^m (c+dx)^n.m"

---

Test results for the 78 problems in "1.1.1.6 P(x) (a+bx)^m (c+dx)^n (e+fx)^p.m"

---

Test results for the 35 problems in "1.1.1.7 P(x) (a+bx)^m (c+dx)^n (e+fx)^p (g+hx)^q.m"

---

Test results for the 1071 problems in "1.1.2.2 (cx)^m (a+bx^2)^p.m"

Problem 662: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left( \frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}} + \frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}} \right) dx$$

Optimal (type 3, 17 leaves, ? steps):

$$x^{2+m} \sqrt{a+bx^2}$$

Result (type 5, 127 leaves, 5 steps):

$$\frac{a x^{2+m} \sqrt{1 + \frac{bx^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\frac{bx^2}{a}\right]}{\sqrt{a+bx^2}} + \frac{b(3+m)x^{4+m} \sqrt{1 + \frac{bx^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4+m}{2}, \frac{6+m}{2}, -\frac{bx^2}{a}\right]}{(4+m)\sqrt{a+bx^2}}$$

**Problem 664: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( -\frac{bx^{1+m}}{(a+bx^2)^{3/2}} + \frac{mx^{-1+m}}{\sqrt{a+bx^2}} \right) dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+bx^2}}$$

Result (type 5, 123 leaves, 5 steps):

$$\frac{x^m \sqrt{1 + \frac{bx^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{m}{2}, \frac{2+m}{2}, -\frac{bx^2}{a}\right]}{\sqrt{a+bx^2}} - \frac{bx^{2+m} \sqrt{1 + \frac{bx^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\frac{bx^2}{a}\right]}{a(2+m)\sqrt{a+bx^2}}$$

---

**Test results for the 349 problems in "1.1.2.3 (a+bx^2)^p (c+dx^2)^q.m"**

**Problem 301: Result unnecessarily involves higher level functions.**

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}} dx$$

Optimal (type 5, 62 leaves, ? steps):

$$\frac{2^{-2-m} \sqrt{x^2} (2 - 4x^2)^{1+m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, (1 - 2x^2)^2\right]}{(1+m)x}$$

Result (type 6, 23 leaves, 1 step):

$$x \text{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 2x^2, x^2\right]$$

Test results for the 1156 problems in "1.1.2.4 (e x)^m (a+b x^2)^p (c+d x^2)^q.m"

Test results for the 115 problems in "1.1.2.5 (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r.m"

Test results for the 51 problems in "1.1.2.6 (g x)^m (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r.m"

Test results for the 174 problems in "1.1.2.8 P(x) (c x)^m (a+b x^2)^p.m"

Test results for the 3078 problems in "1.1.3.2 (c x)^m (a+b x^n)^p.m"

Problem 2686: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left( -\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}} + \frac{mx^{-1+m}}{\sqrt{a+bx^n}} \right) dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+bx^n}}$$

Result (type 5, 126 leaves, 5 steps):

$$\frac{x^m \sqrt{1 + \frac{bx^n}{a}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{m}{n}, \frac{m+n}{n}, -\frac{bx^n}{a}\right]}{\sqrt{a+bx^n}} - \frac{bnx^{m+n} \sqrt{1 + \frac{bx^n}{a}} \text{Hypergeometric2F1}\left[\frac{3}{2}, \frac{m+n}{n}, 2 + \frac{m}{n}, -\frac{bx^n}{a}\right]}{2a(m+n)\sqrt{a+bx^n}}$$

Problem 2697: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left( \frac{6 a x^2}{b (4+m) \sqrt{a+b x^{-2+m}}} + \frac{x^m}{\sqrt{a+b x^{-2+m}}} \right) dx$$

Optimal (type 3, 26 leaves, ? steps):

$$\frac{2 x^3 \sqrt{a+b x^{-2+m}}}{b (4+m)}$$

Result (type 5, 160 leaves, 5 steps):

$$\frac{2 a x^3 \sqrt{1 + \frac{b x^{-2+m}}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{3}{2-m}, -\frac{1+m}{2-m}, -\frac{b x^{-2+m}}{a}\right]}{b (4+m) \sqrt{a+b x^{-2+m}}} + \frac{x^{1+m} \sqrt{1 + \frac{b x^{-2+m}}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{1+m}{2-m}, \frac{1-2m}{2-m}, -\frac{b x^{-2+m}}{a}\right]}{(1+m) \sqrt{a+b x^{-2+m}}}$$

Problem 2699: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left( -\frac{b n x^{-1+m+n}}{2 (a+b x^n)^{3/2}} + \frac{m x^{-1+m}}{\sqrt{a+b x^n}} \right) dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b x^n}}$$

Result (type 5, 126 leaves, 5 steps):

$$\frac{x^m \sqrt{1 + \frac{b x^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{m}{n}, \frac{m+n}{n}, -\frac{b x^n}{a}\right]}{\sqrt{a+b x^n}} - \frac{b n x^{m+n} \sqrt{1 + \frac{b x^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{m+n}{n}, 2 + \frac{m}{n}, -\frac{b x^n}{a}\right]}{2 a (m+n) \sqrt{a+b x^n}}$$

Test results for the 385 problems in "1.1.3.3 (a+b x^n)^p (c+d x^n)^q.m"

Test results for the 1081 problems in "1.1.3.4 (e x)^m (a+b x^n)^p (c+d x^n)^q.m"

Problem 455: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{(8 c - d x^3)^2 (c + d x^3)^{3/2}} dx$$

Optimal (type 4, 256 leaves, ? steps):

$$\frac{2x(4c+dx^3)}{81cd^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{2\sqrt{2+\sqrt{3}}(c^{1/3}+d^{1/3}x)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{((1+\sqrt{3})c^{1/3}+d^{1/3}x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3})c^{1/3}+d^{1/3}x}{(1+\sqrt{3})c^{1/3}+d^{1/3}x}\right], -7-4\sqrt{3}\right]}{81 \times 3^{1/4} c d^{7/3} \sqrt{\frac{c^{1/3}(c^{1/3}+d^{1/3}x)}{((1+\sqrt{3})c^{1/3}+d^{1/3}x)^2}} \sqrt{c+dx^3}}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^7 \sqrt{1 + \frac{dx^3}{c}} \operatorname{AppellF1}\left[\frac{7}{3}, 2, \frac{3}{2}, \frac{10}{3}, \frac{dx^3}{8c}, -\frac{dx^3}{c}\right]}{448c^3 \sqrt{c+dx^3}}$$

Test results for the 46 problems in "1.1.3.6 (g x)^m (a+b x^n)^p (c+d x^n)^q (e+f x^n)^r.m"

Test results for the 594 problems in "1.1.3.8 P(x) (c x)^m (a+b x^n)^p.m"

Test results for the 454 problems in "1.1.4.2 (c x)^m (a x^j+b x^n)^p.m"

Test results for the 298 problems in "1.1.4.3 (e x)^m (a x^j+b x^k)^p (c+d x^n)^q.m"

Test results for the 143 problems in "1.2.1.1 (a+b x+c x^2)^p.m"

Test results for the 2590 problems in "1.2.1.2 (d+e x)^m (a+b x+c x^2)^p.m"

Test results for the 2646 problems in "1.2.1.3 (d+e x)^m (f+g x) (a+b x+c x^2)^p.m"

Test results for the 958 problems in "1.2.1.4 (d+e x)^m (f+g x)^n (a+b x+c x^2)^p.m"

Problem 833: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{-1+x} \sqrt{1+x}}{1+x-x^2} dx$$

Optimal (type 3, 91 leaves, ? steps):

$$-\text{ArcCosh}[x] + \sqrt{\frac{2}{5}(-1 + \sqrt{5})} \text{ArcTan}\left[\frac{\sqrt{1+x}}{\sqrt{-2+\sqrt{5}}\sqrt{-1+x}}\right] + \sqrt{\frac{2}{5}(1 + \sqrt{5})} \text{ArcTanh}\left[\frac{\sqrt{1+x}}{\sqrt{2+\sqrt{5}}\sqrt{-1+x}}\right]$$

Result (type 3, 191 leaves, 9 steps):

$$\frac{\sqrt{\frac{1}{10}(-1 + \sqrt{5})} \sqrt{-1+x} \sqrt{1+x} \text{ArcTan}\left[\frac{2-(1-\sqrt{5})x}{\sqrt{2(-1+\sqrt{5})}\sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}} - \frac{\sqrt{-1+x} \sqrt{1+x} \text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right] - \sqrt{\frac{1}{10}(1 + \sqrt{5})} \sqrt{-1+x} \sqrt{1+x} \text{ArcTanh}\left[\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})}\sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}}$$

---

Test results for the 123 problems in "1.2.1.5 (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

---

Test results for the 143 problems in "1.2.1.6 (g+h x)^m (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

---

Test results for the 400 problems in "1.2.1.9 P(x) (d+e x)^m (a+b x+c x^2)^p.m"

---

Test results for the 1126 problems in "1.2.2.2 (d x)^m (a+b x^2+c x^4)^p.m"

---

Test results for the 413 problems in "1.2.2.3 (d+e x^2)^m (a+b x^2+c x^4)^p.m"

Problem 174: Unable to integrate problem.

$$\int \frac{\sqrt{a + b x^2}}{\sqrt{1 - x^4}} dx$$

Optimal (type 4, 112 leaves, ? steps):



$$\frac{a \sqrt{1-x^2} \sqrt{\frac{a(1+x^2)}{a+bx^2}} \operatorname{EllipticPi}\left[\frac{b}{a+b}, \operatorname{ArcSin}\left[\frac{\sqrt{a+bx}}{\sqrt{a+bx^2}}\right], -\frac{a-b}{a+b}\right]}{\sqrt{a+b} \sqrt{1+x^2} \sqrt{\frac{a(1-x^2)}{a+bx^2}}}$$

Result (type 8, 25 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}, x\right]$$

## Test results for the 413 problems in "1.2.2.4 (f x)^m (d+e x^2)^q (a+b x^2+c x^4)^p.m"

Problem 374: Result valid but suboptimal antiderivative.

$$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)} dx$$

Optimal (type 3, 260 leaves, ? steps):

$$-\frac{d\sqrt{d+ex^2}}{ax} - \frac{(2cd - (b - \sqrt{b^2 - 4ac})e)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}x}{\sqrt{b - \sqrt{b^2 - 4ac}}\sqrt{d+ex^2}}\right]}{\sqrt{b^2 - 4ac} (b - \sqrt{b^2 - 4ac})^{3/2}} + \frac{(2cd - (b + \sqrt{b^2 - 4ac})e)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})e}x}{\sqrt{b + \sqrt{b^2 - 4ac}}\sqrt{d+ex^2}}\right]}{\sqrt{b^2 - 4ac} (b + \sqrt{b^2 - 4ac})^{3/2}}$$

Result (type 3, 432 leaves, 16 steps):

$$\begin{aligned}
& - \frac{d \sqrt{d+ex^2}}{ax} - \frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})} e \left( d + \frac{bd - 2ae}{\sqrt{b^2 - 4ac}} \right) \text{ArcTan} \left[ \frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})} e x}{\sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{d+ex^2}} \right]}{2a \sqrt{b - \sqrt{b^2 - 4ac}}} \\
& + \frac{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})} e \left( d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}} \right) \text{ArcTan} \left[ \frac{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})} e x}{\sqrt{b + \sqrt{b^2 - 4ac}} \sqrt{d+ex^2}} \right]}{2a \sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{d \sqrt{e} \text{ArcTanh} \left[ \frac{\sqrt{e} x}{\sqrt{d+ex^2}} \right]}{a} \\
& - \frac{\sqrt{e} \left( d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}} \right) \text{ArcTanh} \left[ \frac{\sqrt{e} x}{\sqrt{d+ex^2}} \right]}{2a} - \frac{\sqrt{e} \left( d + \frac{bd - 2ae}{\sqrt{b^2 - 4ac}} \right) \text{ArcTanh} \left[ \frac{\sqrt{e} x}{\sqrt{d+ex^2}} \right]}{2a}
\end{aligned}$$

Test results for the 111 problems in "1.2.2.5 P(x) (a+bx^2+cx^4)^p.m"

Test results for the 145 problems in "1.2.2.6 P(x) (dx)^m (a+bx^2+cx^4)^p.m"

Test results for the 42 problems in "1.2.2.7 P(x) (d+ex^2)^q (a+bx^2+cx^4)^p.m"

Test results for the 4 problems in "1.2.2.8 P(x) (d+ex)^q (a+bx^2+cx^4)^p.m"

Test results for the 664 problems in "1.2.3.2 (dx)^m (a+bx^n+cx^(2n))^p.m"

Problem 24: Result valid but suboptimal antiderivative.

$$\int x^8 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Optimal (type 2, 119 leaves, ? steps):

$$\frac{a^2 (a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{12b^3} - \frac{2a (a + bx^3)^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{15b^3} + \frac{(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{18b^3}$$

Result (type 2, 167 leaves, 4 steps):

$$\frac{a^3 x^9 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{9 (a + b x^3)} + \frac{a^2 b x^{12} \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{4 (a + b x^3)} + \frac{a b^2 x^{15} \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{5 (a + b x^3)} + \frac{b^3 x^{18} \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{18 (a + b x^3)}$$

Problem 478: Result unnecessarily involves higher level functions.

$$\int \left( \frac{(a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{x^2} - \frac{2 b^3 (1 - 2 p) (1 - p) p (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{3 a^3 x} \right) dx$$

Optimal (type 3, 146 leaves, ? steps):

$$-\frac{(a + b x^{1/3}) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{a x} + \frac{b (1 - p) (a + b x^{1/3}) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{a^2 x^{2/3}} - \frac{b^2 (1 - 2 p) (1 - p) (a + b x^{1/3}) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{a^3 x^{1/3}}$$

Result (type 5, 162 leaves, 7 steps):

$$\frac{1}{a^3 (1 + 2 p)} 2 b^3 (1 - 2 p) (1 - p) p \left( 1 + \frac{b x^{1/3}}{a} \right) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p \text{Hypergeometric2F1} \left[ 1, 1 + 2 p, 2 (1 + p), 1 + \frac{b x^{1/3}}{a} \right] + \frac{3 b^3 \left( 1 + \frac{b x^{1/3}}{a} \right) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p \text{Hypergeometric2F1} \left[ 4, 1 + 2 p, 2 (1 + p), 1 + \frac{b x^{1/3}}{a} \right]}{a^3 (1 + 2 p)}$$

Test results for the 96 problems in "1.2.3.3 (d+e x^n)^q (a+b x^n+c x^(2 n))^p.m"

Test results for the 156 problems in "1.2.3.4 (f x)^m (d+e x^n)^q (a+b x^n+c x^(2 n))^p.m"

Test results for the 17 problems in "1.2.3.5 P(x) (d x)^m (a+b x^n+c x^(2 n))^p.m"

Test results for the 140 problems in "1.2.4.2 (d x)^m (a x^q+b x^n+c x^(2 n-q))^p.m"

Test results for the 494 problems in "1.3.1 Rational functions.m"

Problem 174: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b x^{1+p} (b x + c x^3)^p + 2 c x^{3+p} (b x + c x^3)^p) dx$$

Optimal (type 3, 27 leaves, ? steps):

$$\frac{x^{1+p} (bx + cx^3)^{1+p}}{2(1+p)}$$

Result (type 5, 116 leaves, 7 steps):

$$\frac{bx^{2+p} \left(1 + \frac{cx^2}{b}\right)^{-p} (bx + cx^3)^p \operatorname{Hypergeometric2F1}\left[-p, 1+p, 2+p, -\frac{cx^2}{b}\right]}{2(1+p)} + \frac{cx^{4+p} \left(1 + \frac{cx^2}{b}\right)^{-p} (bx + cx^3)^p \operatorname{Hypergeometric2F1}\left[-p, 2+p, 3+p, -\frac{cx^2}{b}\right]}{2+p}$$

**Problem 221: Result valid but suboptimal antiderivative.**

$$\int (1+2x)(x+x^2)^3(-18+7(x+x^2)^3)^2 dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81x^4(1+x)^4 - 36x^7(1+x)^7 + \frac{49}{10}x^{10}(1+x)^{10}$$

Result (type 1, 96 leaves, 3 steps):

$$81x^4 + 324x^5 + 486x^6 + 288x^7 - 171x^8 - 756x^9 - \frac{12551x^{10}}{10} - 1211x^{11} - \frac{1071x^{12}}{2} + 336x^{13} + 993x^{14} + \frac{6174x^{15}}{5} + 1029x^{16} + 588x^{17} + \frac{441x^{18}}{2} + 49x^{19} + \frac{49x^{20}}{10}$$

**Problem 222: Result valid but suboptimal antiderivative.**

$$\int x^3(1+x)^3(1+2x)(-18+7x^3(1+x)^3)^2 dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81x^4(1+x)^4 - 36x^7(1+x)^7 + \frac{49}{10}x^{10}(1+x)^{10}$$

Result (type 1, 96 leaves, 2 steps):

$$81x^4 + 324x^5 + 486x^6 + 288x^7 - 171x^8 - 756x^9 - \frac{12551x^{10}}{10} - 1211x^{11} - \frac{1071x^{12}}{2} + 336x^{13} + 993x^{14} + \frac{6174x^{15}}{5} + 1029x^{16} + 588x^{17} + \frac{441x^{18}}{2} + 49x^{19} + \frac{49x^{20}}{10}$$

### Problem 329: Result valid but suboptimal antiderivative.

$$\int \frac{-20x + 4x^2}{9 - 10x^2 + x^4} dx$$

Optimal (type 3, 31 leaves, ? steps):

$$\text{Log}[1 - x] - \frac{1}{2} \text{Log}[3 - x] + \frac{3}{2} \text{Log}[1 + x] - 2 \text{Log}[3 + x]$$

Result (type 3, 41 leaves, 11 steps):

$$-\frac{3}{2} \text{ArcTanh}\left[\frac{x}{3}\right] + \frac{\text{ArcTanh}[x]}{2} + \frac{5}{4} \text{Log}[1 - x^2] - \frac{5}{4} \text{Log}[9 - x^2]$$

### Problem 393: Unable to integrate problem.

$$\int \frac{(1 + x^2)^2}{ax^6 + b(1 + x^2)^3} dx$$

Optimal (type 3, 168 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3} + b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{a^{1/3} + b^{1/3}} b^{5/6}} + \frac{\text{ArcTan}\left[\frac{\sqrt{-(-1)^{1/3} a^{1/3} + b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{-(-1)^{1/3} a^{1/3} + b^{1/3}} b^{5/6}} + \frac{\text{ArcTan}\left[\frac{\sqrt{(-1)^{2/3} a^{1/3} + b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{(-1)^{2/3} a^{1/3} + b^{1/3}} b^{5/6}}$$

Result (type 8, 68 leaves, 5 steps):

$$\text{CannotIntegrate}\left[\frac{1}{ax^6 + b(1 + x^2)^3}, x\right] + 2 \text{CannotIntegrate}\left[\frac{x^2}{ax^6 + b(1 + x^2)^3}, x\right] + \text{CannotIntegrate}\left[\frac{x^4}{ax^6 + b(1 + x^2)^3}, x\right]$$

### Problem 493: Unable to integrate problem.

$$\int \left( \frac{3(-47 + 228x + 120x^2 + 19x^3)}{(3 + x + x^4)^4} + \frac{42 - 320x - 75x^2 - 8x^3}{(3 + x + x^4)^3} + \frac{30x}{(3 + x + x^4)^2} \right) dx$$

Optimal (type 1, 27 leaves, ? steps):

$$\frac{2 - 3x + 5x^2 + x^4 - 5x^6}{(3 + x + x^4)^3}$$

Result (type 8, 121 leaves, 7 steps):

$$\begin{aligned}
& - \frac{19}{4(3+x+x^4)^3} + \frac{1}{(3+x+x^4)^2} - \frac{621}{4} \text{CannotIntegrate}\left[\frac{1}{(3+x+x^4)^4}, x\right] + \\
& 684 \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^4}, x\right] + 360 \text{CannotIntegrate}\left[\frac{x^2}{(3+x+x^4)^4}, x\right] + 44 \text{CannotIntegrate}\left[\frac{1}{(3+x+x^4)^3}, x\right] - \\
& 320 \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^3}, x\right] - 75 \text{CannotIntegrate}\left[\frac{x^2}{(3+x+x^4)^3}, x\right] + 30 \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^2}, x\right]
\end{aligned}$$

### Problem 494: Unable to integrate problem.

$$\int \left( \frac{-3 + 10x + 4x^3 - 30x^5}{(3+x+x^4)^3} - \frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4} \right) dx$$

Optimal (type 1, 27 leaves, ? steps):

$$\frac{2 - 3x + 5x^2 + x^4 - 5x^6}{(3+x+x^4)^3}$$

Result (type 8, 177 leaves, 13 steps):

$$\begin{aligned}
& \frac{7}{2(3+x+x^4)^3} - \frac{63x}{22(3+x+x^4)^3} - \frac{12x^2}{(3+x+x^4)^3} - \frac{5x^3}{(3+x+x^4)^3} + \frac{3x^4}{2(3+x+x^4)^3} - \frac{10x^6}{(3+x+x^4)^3} - \\
& \frac{1}{2(3+x+x^4)^2} + \frac{5x^2}{(3+x+x^4)^2} + \frac{144}{11} \text{CannotIntegrate}\left[\frac{1}{(3+x+x^4)^4}, x\right] + \frac{828}{11} \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^4}, x\right] + \\
& 18 \text{CannotIntegrate}\left[\frac{x^2}{(3+x+x^4)^4}, x\right] - 4 \text{CannotIntegrate}\left[\frac{1}{(3+x+x^4)^3}, x\right] - 20 \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^3}, x\right]
\end{aligned}$$

## Test results for the 1025 problems in "1.3.2 Algebraic functions.m"

### Problem 197: Unable to integrate problem.

$$\int \frac{(d^3 + e^3 x^3)^p}{d + ex} dx$$

Optimal (type 6, 135 leaves, ? steps):

$$\frac{(d^3 + e^3 x^3)^p \left(1 + \frac{2(d+ex)}{(-3+i\sqrt{3})d}\right)^{-p} \left(1 - \frac{2(d+ex)}{(3+i\sqrt{3})d}\right)^{-p} \text{AppellF1}\left[p, -p, -p, 1+p, -\frac{2(d+ex)}{(-3+i\sqrt{3})d}, \frac{2(d+ex)}{(3+i\sqrt{3})d}\right]}{e^p}$$

Result (type 8, 23 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{(d^3 + e^3 x^3)^p}{d + e x}, x\right]$$

**Problem 396: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( \frac{\sqrt{a x^{2n}}}{\sqrt{1+x^n}} + \frac{2 x^{-n} \sqrt{a x^{2n}}}{(2+n) \sqrt{1+x^n}} \right) dx$$

Optimal (type 3, 34 leaves, ? steps):

$$\frac{2 x^{1-n} \sqrt{a x^{2n}} \sqrt{1+x^n}}{2+n}$$

Result (type 5, 80 leaves, 5 steps):

$$\frac{x \sqrt{a x^{2n}} \text{Hypergeometric2F1}\left[\frac{1}{2}, 1 + \frac{1}{n}, 2 + \frac{1}{n}, -x^n\right]}{1+n} + \frac{2 x^{1-n} \sqrt{a x^{2n}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{n}, 1 + \frac{1}{n}, -x^n\right]}{2+n}$$

**Problem 616: Unable to integrate problem.**

$$\int \frac{1}{x^2} (a + b x + c x^2)^m (d + e x + f x^2 + g x^3)^n (-a d + (b d m + a e n) x + (c d + b e + a f + 2 c d m + b e m + b e n + 2 a f n) x^2 + (2 c e + 2 b f + 2 a g + 2 c e m + b f m + c e n + 2 b f n + 3 a g n) x^3 + (3 c f + 3 b g + 2 c f m + b g m + 2 c f n + 3 b g n) x^4 + c g (4 + 2 m + 3 n) x^5) dx$$

Optimal (type 3, 37 leaves, ? steps):

$$\frac{(a + b x + c x^2)^{1+m} (d + e x + f x^2 + g x^3)^{1+n}}{x}$$

Result (type 8, 306 leaves, 2 steps):

$$\begin{aligned} & (c (d + 2 d m) + b e (1 + m + n) + a f (1 + 2 n)) \text{CannotIntegrate}\left[(a + b x + c x^2)^m (d + e x + f x^2 + g x^3)^n, x\right] - \\ & a d \text{CannotIntegrate}\left[\frac{(a + b x + c x^2)^m (d + e x + f x^2 + g x^3)^n}{x^2}, x\right] + (b d m + a e n) \text{CannotIntegrate}\left[\frac{(a + b x + c x^2)^m (d + e x + f x^2 + g x^3)^n}{x}, x\right] + \\ & (c e (2 + 2 m + n) + b f (2 + m + 2 n) + a g (2 + 3 n)) \text{CannotIntegrate}\left[x (a + b x + c x^2)^m (d + e x + f x^2 + g x^3)^n, x\right] + \\ & (c f (3 + 2 m + 2 n) + b g (3 + m + 3 n)) \text{CannotIntegrate}\left[x^2 (a + b x + c x^2)^m (d + e x + f x^2 + g x^3)^n, x\right] + \\ & c g (4 + 2 m + 3 n) \text{CannotIntegrate}\left[x^3 (a + b x + c x^2)^m (d + e x + f x^2 + g x^3)^n, x\right] \end{aligned}$$

### Problem 617: Unable to integrate problem.

$$\int \frac{1}{x^3} (a + b x + c x^2)^m (d + e x + f x^2 + g x^3)^n (-2 a d + (-b d - a e + b d m + a e n) x + (2 c d m + b e m + b e n + 2 a f n) x^2 + (c e + b f + a g + 2 c e m + b f m + c e n + 2 b f n + 3 a g n) x^3 + (2 c f + 2 b g + 2 c f m + b g m + 2 c f n + 3 b g n) x^4 + c g (3 + 2 m + 3 n) x^5) dx$$

Optimal (type 3, 37 leaves, ? steps):

$$\frac{(a + b x + c x^2)^{1+m} (d + e x + f x^2 + g x^3)^{1+n}}{x^2}$$

Result (type 8, 305 leaves, 2 steps):

$$\begin{aligned} & (c e (1 + 2 m + n) + b f (1 + m + 2 n) + a g (1 + 3 n)) \text{CannotIntegrate} \left[ (a + b x + c x^2)^m (d + e x + f x^2 + g x^3)^n, x \right] - \\ & 2 a d \text{CannotIntegrate} \left[ \frac{(a + b x + c x^2)^m (d + e x + f x^2 + g x^3)^n}{x^3}, x \right] - \\ & (b d (1 - m) + a e (1 - n)) \text{CannotIntegrate} \left[ \frac{(a + b x + c x^2)^m (d + e x + f x^2 + g x^3)^n}{x^2}, x \right] + \\ & (2 c d m + 2 a f n + b e (m + n)) \text{CannotIntegrate} \left[ \frac{(a + b x + c x^2)^m (d + e x + f x^2 + g x^3)^n}{x}, x \right] + \\ & (2 c f (1 + m + n) + b g (2 + m + 3 n)) \text{CannotIntegrate} \left[ x (a + b x + c x^2)^m (d + e x + f x^2 + g x^3)^n, x \right] + \\ & c g (3 + 2 m + 3 n) \text{CannotIntegrate} \left[ x^2 (a + b x + c x^2)^m (d + e x + f x^2 + g x^3)^n, x \right] \end{aligned}$$

### Problem 941: Result unnecessarily involves higher level functions.

$$\int \left( (1 - x^6)^{2/3} + \frac{(1 - x^6)^{2/3}}{x^6} \right) dx$$

Optimal (type 2, 35 leaves, ? steps):

$$-\frac{(1 - x^6)^{2/3}}{5 x^5} + \frac{1}{5} x (1 - x^6)^{2/3}$$

Result (type 5, 36 leaves, 3 steps):

$$-\frac{\text{Hypergeometric2F1} \left[ -\frac{5}{6}, -\frac{2}{3}, \frac{1}{6}, x^6 \right]}{5 x^5} + x \text{Hypergeometric2F1} \left[ -\frac{2}{3}, \frac{1}{6}, \frac{7}{6}, x^6 \right]$$



### Problem 995: Unable to integrate problem.

$$\int \sqrt{1-x^2+x} \sqrt{-1+x^2} \, dx$$

Optimal (type 3, 63 leaves, ? steps):

$$\frac{1}{4} \left( 3x + \sqrt{-1+x^2} \right) \sqrt{1-x^2+x} \sqrt{-1+x^2} + \frac{3 \operatorname{ArcSin} \left[ x - \sqrt{-1+x^2} \right]}{4\sqrt{2}}$$

Result (type 8, 24 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \sqrt{1-x^2+x} \sqrt{-1+x^2}, x \right]$$

### Problem 996: Unable to integrate problem.

$$\int \frac{\sqrt{-x+\sqrt{x}} \sqrt{1+x}}{\sqrt{1+x}} \, dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2} \left( \sqrt{x} + 3\sqrt{1+x} \right) \sqrt{-x+\sqrt{x}} \sqrt{1+x} - \frac{3 \operatorname{ArcSin} \left[ \sqrt{x} - \sqrt{1+x} \right]}{2\sqrt{2}}$$

Result (type 8, 31 leaves, 1 step):

$$\text{CannotIntegrate} \left[ \frac{\sqrt{-x+\sqrt{x}} \sqrt{1+x}}{\sqrt{1+x}}, x \right]$$

### Problem 997: Result valid but suboptimal antiderivative.

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}} \, dx$$

Optimal (type 3, 78 leaves, ? steps):

$$-\sqrt{2(1+\sqrt{5})} \operatorname{ArcTan} \left[ \sqrt{-2+\sqrt{5}} \left( x + \sqrt{1+x^2} \right) \right] + \sqrt{2(-1+\sqrt{5})} \operatorname{ArcTanh} \left[ \sqrt{2+\sqrt{5}} \left( x + \sqrt{1+x^2} \right) \right]$$

Result (type 3, 319 leaves, 25 steps):

$$\begin{aligned}
& -2 \sqrt{\frac{2}{5(1+\sqrt{5})}} \operatorname{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}} x\right] - \sqrt{\frac{1}{10}(1+\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}} x\right] - \sqrt{\frac{2}{5(-1+\sqrt{5})}} \operatorname{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}} \sqrt{1+x^2}\right] - \\
& \sqrt{\frac{2}{5(-1+\sqrt{5})}} \operatorname{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}} \sqrt{1+x^2}\right] - 2 \sqrt{\frac{2}{5(-1+\sqrt{5})}} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}} x\right] + \\
& \sqrt{\frac{1}{10}(-1+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}} x\right] - \sqrt{\frac{2}{5(1+\sqrt{5})}} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{1+x^2}\right] + \sqrt{\frac{2}{5}(1+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{1+x^2}\right]
\end{aligned}$$

**Problem 1017: Result valid but suboptimal antiderivative.**

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}} dx$$

Optimal (type 3, 103 leaves, ? steps):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1-2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2^{2/3}} - \frac{\operatorname{Log}\left[1+2(1-x)^3-x^3\right]}{2 \times 2^{2/3}} + \frac{3 \operatorname{Log}\left[2^{1/3}(1-x)+(1-x^3)^{1/3}\right]}{2 \times 2^{2/3}}$$

Result (type 3, 425 leaves, 42 steps):

$$\begin{aligned}
& \frac{2^{1/3} \operatorname{ArcTan}\left[\frac{1-2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{\sqrt{3}} + \frac{\operatorname{ArcTan}\left[\frac{1+2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2^{2/3} \sqrt{3}} - \frac{\operatorname{ArcTan}\left[\frac{1-2^{2/3}x}{(1-x^3)^{1/3}}\right]}{2^{2/3} \sqrt{3}} + \frac{\operatorname{ArcTan}\left[\frac{1+2^{2/3}(1-x^3)^{1/3}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}} + \frac{\operatorname{Log}\left[1+x^3\right]}{3 \times 2^{2/3}} + \frac{\operatorname{Log}\left[2^{2/3}-\frac{1-x}{(1-x^3)^{1/3}}\right]}{3 \times 2^{2/3}} - \\
& \frac{\operatorname{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3 \times 2^{2/3}} + \frac{1}{3} \times 2^{1/3} \operatorname{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right] - \frac{\operatorname{Log}\left[2 \times 2^{1/3}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{2/3}} - \frac{\operatorname{Log}\left[2^{1/3}-(1-x^3)^{1/3}\right]}{2 \times 2^{2/3}} - \frac{\operatorname{Log}\left[-2^{1/3}x-(1-x^3)^{1/3}\right]}{2 \times 2^{2/3}}
\end{aligned}$$

**Problem 1018: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2}{\sqrt{-1+x^4}(1+x^4)} dx$$

Optimal (type 3, 49 leaves, ? steps):

$$-\frac{1}{4} \operatorname{ArcTan}\left[\frac{1+x^2}{x\sqrt{-1+x^4}}\right] - \frac{1}{4} \operatorname{ArcTanh}\left[\frac{1-x^2}{x\sqrt{-1+x^4}}\right]$$

Result (type 3, 47 leaves, 9 steps):

$$\left(-\frac{1}{8} - \frac{i}{8}\right) \text{ArcTan}\left[\frac{(1+i)x}{\sqrt{-1+x^4}}\right] + \left(\frac{1}{8} + \frac{i}{8}\right) \text{ArcTanh}\left[\frac{(1+i)x}{\sqrt{-1+x^4}}\right]$$

Problem 1023: Unable to integrate problem.

$$\int (1+x+x^2+x^3)^{-n} (1-x^4)^n dx$$

Optimal (type 3, 34 leaves, ? steps):

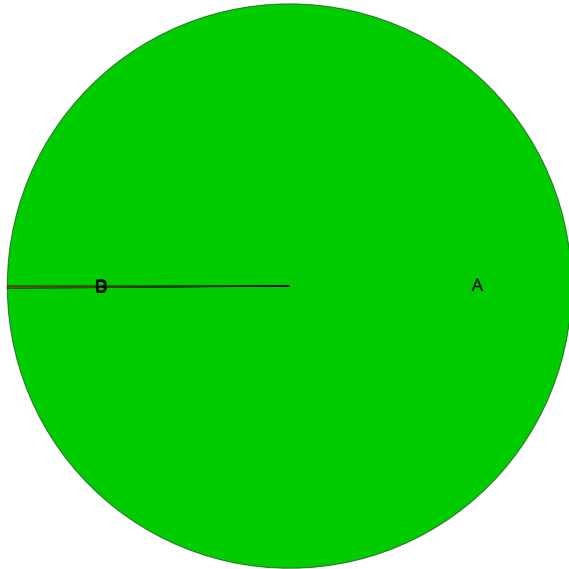
$$\frac{(1-x)(1+x+x^2+x^3)^{-n}(1-x^4)^n}{1+n}$$

Result (type 8, 25 leaves, 0 steps):

$$\text{CannotIntegrate}\left[(1+x+x^2+x^3)^{-n}(1-x^4)^n, x\right]$$

## Summary of Integration Test Results

26125 integration problems



- A - 26092 optimal antiderivatives
- B - 9 valid but suboptimal antiderivatives
- C - 13 unnecessarily complex antiderivatives
- D - 11 unable to integrate problems
- E - 0 integration timeouts
- F - 0 invalid antiderivatives